Title : will be set by the publisher
Editors : will be set by the publisher
EAS Publications Series, Vol. ?, 2008

N-BODY SIMULATIONS IN MODIFIED NEWTONIAN DYNAMICS

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Abstract. We describe some results obtained with N-MODY, a code for N-body simulations of collisionless stellar systems in modified Newtonian dynamics (MOND). We found that a few fundamental dynamical processes are profoundly different in MOND and in Newtonian gravity with dark matter. In particular, violent relaxation, phase mixing and galaxy merging take significantly longer in MOND than in Newtonian gravity, while dynamical friction is more effective in a MOND system than in an equivalent Newtonian system with dark matter.

1 Introduction

Milgrom (1983) proposed that the kinematics of galaxies might be explained without dark matter if one allows for a modification of Newtonian dynamics in the low-acceleration regime. In Bekenstein & Milgrom's (1984) formulation of Milgrom's modified Newtonian dynamics (MOND) Poisson's equation $\nabla^2\phi^{\mathrm{N}} = 4\pi G\rho$ is replaced by the non-relativistic field equation

$$\nabla \cdot \left[\mu\left(\frac{\|\nabla\phi\|}{a_0}\right)\nabla\phi\right] = 4\pi G\rho, \tag{1.1}$$

with boundary conditions $\nabla\phi \to 0$ as $\|\mathbf{x}\| \to \infty$ for a system of finite mass. Here, ϕ^{N} and ϕ are, respectively, the Newtonian and MOND gravitational potentials produced by the density distribution ρ, and $\|...\|$ is the standard Euclidean norm. The interpolating function $\mu(y)$ runs smoothly from $\mu(y) \sim y$ at $y \ll 1$ to $\mu(y) \sim 1$ at $y \gg 1$, with the transition taking place at $y \approx 1$, i.e., when $\|\nabla\phi\|$ is of order the characteristic acceleration $a_0 \simeq 1.2 \times 10^{-10}\mathrm{m\,s^{-2}}$. From Poisson's equation and equation (1.1) it follows that the MOND gravitational field $\mathbf{g} = -\nabla\phi$ is related to the Newtonian field $\mathbf{g}^{\mathrm{N}} = -\nabla\phi^{\mathrm{N}}$ by $\mu(g/a_0)\,\mathbf{g} = \mathbf{g}^{\mathrm{N}} + \mathbf{S}$, where $g \equiv \|\mathbf{g}\|$, and $\mathbf{S}$

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DOI: (will be inserted later)

is a solenoidal field dependent on the specific ρ considered. Unless the system has special symmetries, $\mathbf{S} \neq 0$ (Bekenstein & Milgrom 1984; Brada & Milgrom 1995).

Due to the non-linearity of MOND, the problem of calculating the gravitational field produced by a distribution of N particles is harder in MOND than in Newtonian dynamics. In MOND there is not an analytic expression for the force between two particles, so to study the two-body problem one must solve equation (1.1) with source term given by the two particles (Milgrom 1986). Similarly, when computing the MOND gravitational potential of a distribution of N particles, one cannot use methods that exploit the linearity of Poisson's equation. Thus, direct summation or hierarchical (Barnes & Hut 1986) Poisson solvers cannot be used in MOND N-body codes. As the solenoidal field $\mathbf{S}$ is typically relatively small for equilibrium configurations (Brada & Milgrom 1995; Ciotti *et al.* 2006), one could be tempted to run N-body simulations assuming $\mathbf{S} = 0$. But neglecting $\mathbf{S}$ when simulating time-dependent dynamical processes has dramatic effects such as non-conservation of total linear momentum, as first pointed out by Felten (1984; see also Nipoti *et al.* 2007a). Instead, equation (1.1) must be solved at each time step. For collisionless systems a natural choice is to consider particle-mesh N-body codes based on a MOND Poisson solver.

2 N-MODY: a code for collisionless N-body simulations in MOND

We developed N-MODY,[1] a parallel three-dimensional particle-mesh code for collisionless N-body simulations in MOND. The N-body code and the potential solver on which the code is based are described in detail in Londrillo & Nipoti (2008), and have been tested and applied in Ciotti *et al.* (2006, 2007) and Nipoti *et al.* (2007a, 2007b, 2007c, 2008). The potential solver of N-MODY solves the MOND field equation (1.1) using a relaxation method in spherical coordinates based on spherical harmonics expansion. Thus the code is ideally suited for simulations of isolated stellar systems (but can be adapted to run simulations of interacting galaxies; see Nipoti *et al.* 2007c). N-MODY is one of the very few MOND N-body codes developed so far: as far as we know the only other three-dimensional MOND N-body code for simulation of collisionless stellar systems is Brada & Milgrom's (1999) code, which is based on a multi-grid potential solver in Cartesian coordinates and has been implemented also by Tiret & Combes (2007). Recently, a code for MOND cosmological N-body simulations has been developed by Llinares *et al.* (2008).

3 Results of N-body simulations in MOND

We have applied N-MODY to study a few relevant stellar dynamical processes, such as collisionless collapse, galaxy merging and dynamical friction.

Our simulations of collisionless collapses (Nipoti *et al.* 2007a) showed that the phase-mixing and violent-relaxation timescales are significantly longer in MOND

[1] The code is publicly available upon request to the Authors.

than in Newtonian gravity (see also Ciotti *et al.* 2007). Remarkably, when MOND systems eventually reach equilibrium, they have projected surface mass density and velocity profiles consistent with observations of elliptical galaxies. However, we found that the collapse end-products cannot satisfy simultaneously the observed Faber & Jackson (1976), Kormendy (1977) and Fundamental Plane (Djorgovski & Davis 1987, Dressler *et al.* 1987) relations of elliptical galaxies, under the assumption of luminosity-independent stellar mass-to-light ratio.

Our results on phase mixing and violent relaxation in MOND suggested that galaxy merging could be less effective in MOND than in Newtonian gravity, also because in MOND galaxies are expected to collide at high speed, and there are no dark matter halos to absorb orbital energy and angular momentum (Binney 2004; Sellwood 2004). Our MOND simulations of collisionless merging confirmed that the merging timescales are significantly longer in MOND than in Newtonian gravity with dark matter, suggesting that observational evidence of rapid merging could be difficult to explain in MOND (Nipoti *et al.* 2007c).

Finally, we applied N-MODY to study dynamical friction in MOND (Nipoti *et al.* 2008). Our simulations showed that the dynamical friction timescale is significantly shorter in MOND systems than in Newtonian systems *with the same phase-space distribution of baryons and additional dark matter*, confirming the analytic estimate of Ciotti & Binney (2004). In Nipoti *et al.* (2008) we explored the case of the evolution a rigid massive bar rotating at the centre of a much more massive stellar system, but our results are expected to apply also to the case of a small satellite orbiting within a stellar system, such as a globular cluster in a galaxy. As a consequence, in the context of MOND it is difficult to explain that the globular clusters of the dwarf spheroidal galaxy Fornax have not sunk yet to the galaxy centre (see also Ciotti & Binney 2004 and Sánchez-Salcedo *et al.* 2006).

4 Concluding remarks

The results of our N-body simulations show that MOND systems evolve dynamically very differently from equivalent Newtonian systems with dark matter. In particular, phase mixing, violent relaxation and galaxy merging are less effective, while dynamical friction is more effective in MOND than in Newtonian gravity with dark matter.

It might seem counterintuitive that in MOND galaxy merging timescales are long, while dynamical friction timescales are short. However, galaxy merging cannot be described purely in terms of dynamical friction. The orbital energy and angular momentum are absorbed mainly by dark matter halos in the case of Newtonian merging, while by the outer parts of the baryonic distributions in MOND merging. Thus, in the Newtonian case there is plenty of particles able to store the orbital energy of the systems, and the concept of dynamical friction does apply, while in the MOND case the particles responsible for the absorption are relatively few, and dynamical friction does not play a central role (Nipoti *et al.* 2008).

Similar considerations apply to the case of a bar rotating at the centre of a

MOND galaxy. For the concept of dynamical friction to apply the bar must rotate in a much more massive stellar system. This is typically the case in Newtonian systems, because of the presence of dark matter halos, but not necessarily in MOND: *if the bar mass is a substantial fraction of the galactic baryonic mass* it might well be that the slowing-down is slower in MOND than in an equivalent Newtonian system with dark matter (as actually found by Tiret & Combes 2007).

These examples show how a careful comparison between equivalent systems is fundamental to draw conclusions about dynamical processes in different gravity theories. This kind of comparison was the aim of our N-body simulations, which actually showed that MOND systems and Newtonian systems with dark matter can be distinguished studying their time evolution. The main result of our experiments is that some phenomena, such as rapid dissipationless galaxy mergers or the survival of globular clusters in dwarf spheroidal galaxies, appear difficult to explain in the context of MOND.

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